perturbation differential equations. The cases mentioned in the last section will now be discussed.

$\mu < 1, k = 1$

In this case the expansions for velocity and pressure are uniformly valid. The first of Eqs. (7) and the last of Eqs. (5) can be used to show that

$$V_1(\eta_{b0}) = (1 - L)B_1$$

and

$$B_1 = 1 - \mu \tag{8}$$

Once B_1 is known, A_1, C_1, and a_1 can be determined as in Mirels' work.[2] However, the expansion for density is not uniformly valid; namely, in an inner region where θ is $0(t^{-N})$, the second term in the expansion D_1t^{-N} attains the same order of magnitude as the first term D_0. Using the method of matched asymptotic expansions, one can show that in the inner region

$$\rho = (C_d/t^{\sigma N}) \times \left[(\bar{\theta} - 1)^\sigma + C_1 \frac{(\bar{\theta} - 1)^{\sigma - \mu}}{t^{(1-\mu)N}} + \ldots \right] \qquad \bar{\theta} = t^N\theta \quad (9)$$

which is still not valid in the innermost region where $(\bar{\theta} - 1)$ is $0(t \exp\{-[(1 - \mu)N/\mu]\})$. In this innermost region the density cannot be determined by the governing equations because it depends on the initial history of the shock; however, given the initial shock motion and using the particle-isentropic condition, the complete asymptotic flow field can be determined. Such dependence of density on the initial history of the shock may happen in the inner region in other cases.

$\mu = 1, k = 1$

Although the asymptotic behavior of V_1 near the piston will be different from the preceding case, with logarithmic terms appearing, it can be proved that the log term is not the dominating term at $\eta = \eta_{b0}$ and, therefore, still $B_1 = 1 - \mu$ which is zero in this case. It follows that this case yields only trivial solutions.

$\mu = 1, k = 2$

The expansion for velocity is still uniformly valid and, as in preceding cases, B_k can be shown to be

$$B_2 = -j/2\eta_{b0} \tag{10}$$

It is obvious that if $j = 0$ the preceding is a trivial solution. For $j = 1$ or 2, A_2, C_2, and a_2 can again be determined. The outer expansion for density is still not valid in the inner region $\theta = 0(t^{-N})$ where now the density already depends on the initial shock motion. The outer expansion for pressure may or may not be uniformly valid depending on whether σ is larger or smaller than unity.

$\mu = 1, j = 0, \sigma < 1, k = 3$

All the expansions of Eqs. (3) are not valid near the piston, and the last of Eqs. (5) can no longer be used to determine A_k, B_k, C_k, and a_k as before. They can only be determined from matching with the inner solutions.

References

[1] Mirels, H., "Hypersonic flow over slender bodies associated with power-law shocks," *Advances in Applied Mechanics* (Academic Press, New York, 1962), Vol. 7, pp. 1–54.

[2] Mirels, H., "Effect of body perturbations on hypersonic flow over slender power law bodies," NASA TR R-45 (1959).

[3] Van Dyke, M. D., "Perturbation methods in fluid mechanics" (Academic Press, New York, 1964).

[4] Giuraud, J. P., "Asymptotic theory in hypersonic flow," Office National D'Études et de Recherches Aerospatiales TP 132 (1964).

[5] Messiter, A. F., "Asymptotic theory of inviscid hypersonic flow at large distance from a blunt-nosed body," Univ. of Michigan BAMIRAC Rept. 4613-81-T (1965).

[6] Vaglio-Laurin, R., "Asymptotic flow pattern of a hypersonic body," Polytechnic Institute of Brooklyn Aerospace Labs., PIBAL Rept. 805 (1964).

[7] Van Dyke, M. D., private communication, Douglas Aircraft Co., Santa Monica, Calif. (1964).

Some Experimental Observations on the Nonlinear Vibration of Cylindrical Shells

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Nomenclature

h = shell thickness (in.)
L = unsupported length of shell (in.)
n = number of circumferential waves in shell vibration mode
p_0 = amplitude of Jensen driver output (psi)
R = shell radius (in.)
V = voltage input to Jensen driver (v rms)
δ = driver-to-shell (or pressure transducer) spacing (in.)

Introduction

In a note on the nonlinear vibration of cylindrical shells, Evensen[1] indicated that, contrary to the findings of Chu[2] and Nowinski,[3] the nonlinearity revealed by his preliminary investigations was of the "softening" type, and the vibrations were only slightly nonlinear. In subsequent work, Evensen[4] treated the analogous problem for a thin-walled ring in great detail both theoretically and experimentally. He found that the ring vibrations exhibited only a small "softening" type of nonlinearity. As a consequence of the similarity between the two problems, these findings lend support to the earlier predictions. However, at the present time, there seems to be no quantitative experimental data available to substantiate these predictions for the complete cylindrical shell.

In preparing for flutter experiments,[5] the author performed vibration tests on several cylindrical shells. The results of these tests included some qualitative observations and some quantitative data on large amplitude vibrations, and these are reported herein.

Experimental

The forementioned shells were thin-walled seamless circular cylinders made of copper by an electroplating process. The large amplitude vibration test was carried out on a 0.0044-in. thick shell (radius-to-thickness ratio of 1820) mounted on the flutter model. Motion of the shell skin was measured with an inductance pickup that could be traversed both axially and circumferentially under the shell without touching it. Full details of the shells and flutter model are reported elsewhere.[5]

The shell vibrations were driven by a Jensen model D-40 acoustic driver whose acoustic output was focused through a conical nozzle with a 0.25-in.-diam exit hole. The driver was positioned midway between the ends of the shell with the

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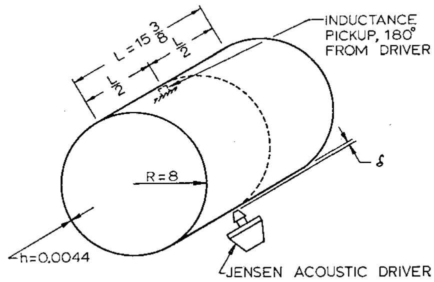

Fig. 1 Scheme of shell vibration setup.

nozzle a small distance δ away from the shell skin (Fig. 1).

The $n = 10$ vibration mode with one axial half-wave (hereafter referred to as the $n = 10$ mode) was one of the cleanest obtained and consequently was chosen for the nonlinear testing. Its natural frequency of vibration was 131.2 cps (Fig. 2).

The Jensen driver was calibrated for this frequency with one of the pressure transducers developed by Schmidt.[6] The pressure output from the driver was essentially sinusoidal for driver-to-transducer spacings δ up to 0.03 in., and the amplitude fell off gradually with increasing δ.

Results

The amplitude responses for the $n = 10$ mode are plotted in Fig. 3, and the corresponding circumferential mode shapes at peak response are shown in Fig. 4. The well-known jump phenomena associated with nonlinear vibrations are clearly exhibited in Fig. 3. For example, as the frequency was increased from below resonance, the vibration amplitude increased slowly until at some critical frequency it suddenly jumped to a high level. At the same time, the phase angle between the vibration and the forcing function jumped from less than 90° to greater than 90°. A further increase in frequency caused a gradual decrease in vibration amplitude and a steady increase in phase angle. On the other hand, when the frequency was then decreased, the vibration amplitude increased until a maximum was reached with a phase angle very close to 90°. A further decrease in frequency then caused the vibration amplitude to jump down to a low value and the phase angle to jump to less than 90°. This second critical frequency was lower than the first one at which the amplitude had jumped up. This shows clearly that the nonlinearity is of the "softening" type. The term jump has been used in the foregoing context to mean a finite change over a time period equivalent to about 100 to 300 cycles of shell motion.

Figure 3 shows that, even for vibration amplitudes of 2.5 shell thicknesses, the change in vibration frequency is less than 1%. This seems to contradict the predictions by Chu[2] and Nowinski[3] of 20 to 50% changes. However a comparison with their work is not straight forward since they presented numerical results only for a relatively thick shell. Figure 4 shows that there was a small amount (exaggerated by plotting mean square amplitude) of another mode present with the $n = 10$ mode.

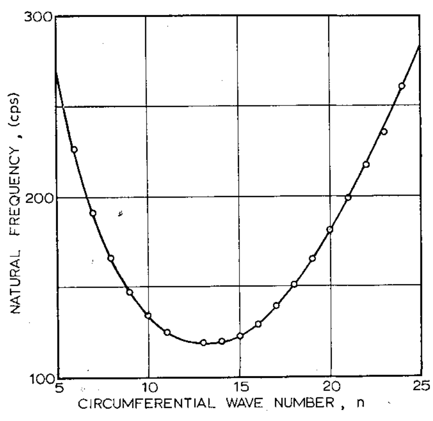

Fig. 2 Natural frequencies for shell modes with one axial half-wave.

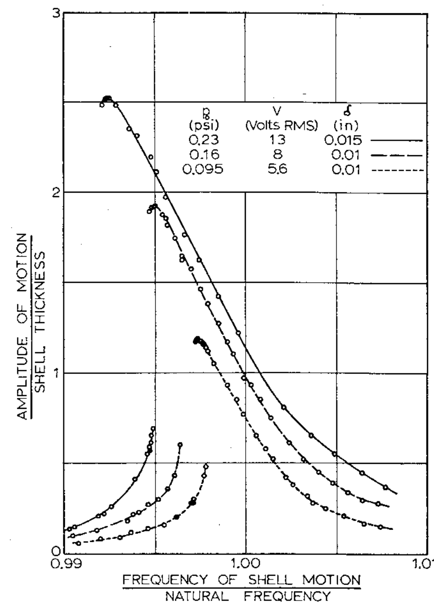

Fig. 3 Shell response in $n = 10$ mode.

It should be noted that the motion of the shell would modulate the effective output of the Jensen driver, since its output varied with the driver-to-shell spacing δ. At the highest amplitude obtained (2.5 shell thickness), this modulation was estimated to be about ±20%, but at lower amplitudes, it would be much less. How much effect this modulation has on the response curves is not known, but it is felt that it would not change the nature of these curves.

Some qualitative observations obtained in the course of this investigation are also of interest here. When any mode of the shell was forced into resonance and the pickup was positioned at a longitudinal nodal line, the Lissajous figure for the vibration was a horizontal figure 8 of relatively small height. This is evidence of the presence of the so-called double frequency nodal contraction found in the ring vibrations.

Ultraharmonics up to $\frac{1}{8}$ were easily obtained for many modes. For example, with the driver oscillating at about 16.4 cps, the shell would vibrate in the $n = 10$ mode at a frequency of 131 cps. These ultraharmonics were also observed in the ring vibrations.

It was also observed that, as any mode was driven to ever increasing amplitudes by increasing the driver voltage, at

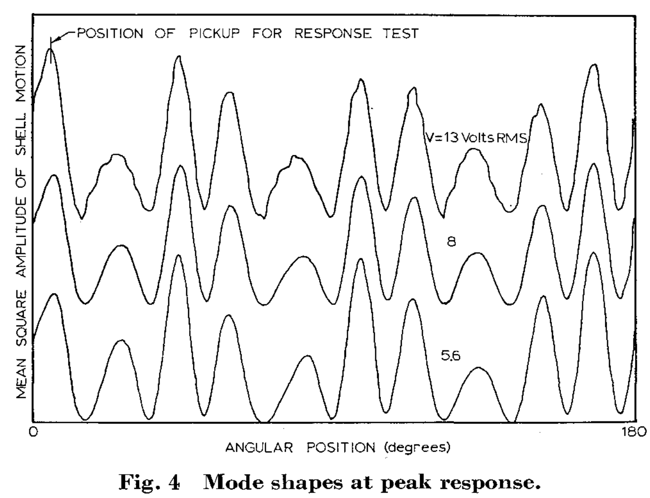

Fig. 4 Mode shapes at peak response.

some critical amplitude the vibration would become unstable. Although no quantitative data were obtained, it was observed that this critical amplitude (which was about 3 shell thicknesses for the $n = 10$ mode) depended on the circumferential wave number n. It is suspected that this is evidence of the occurrence of the companion mode instability observed in the ring vibrations.

Conclusions

The experimental results presented support the thesis that the nonlinear vibrations of thin cylindrical shells in mode shapes with long axial wavelengths exhibit many of the phenomena previously observed for thin circular rings. In particular, for amplitudes of the order of the shell thickness, the vibrations exhibit a slight nonlinearity of the "softening" type.

References

[1] Evensen, D. A., "Some observations on the nonlinear vibration of thin cylindrical shells," AIAA J. **1**, 2857–2858 (1963).

[2] Chu, H. N., "Influence of large amplitudes on flexural vibrations of a thin circular cylindrical shell," J. Aerospace Sci. **28**, 602–609 (1961).

[3] Nowinski, J., "Nonlinear transverse vibrations of orthotropic cylindrical shells," AIAA J. **1**, 617–620 (1963).

[4] Evensen, D. A., "Nonlinear flexural vibrations of thin circular rings," Ph.D. Thesis, California Institute of Technology (June 1964).

[5] Olson, M. D., "Supersonic flutter of circular cylindrical shells subjected to internal pressure and axial compression," Guggenheim Aeronautical Lab., California Institute of Technology Structural Dynamics Rept. SM 65-7, Air Force Office of Scientific Research 65-0599 (April 1965).

[6] Schmidt, L. V., "Measurements of fluctuating air loads on a circular cylinder," Ph.D. Thesis, California Institute of Technology (June 1963).

Electromagnetic Generation of High Dynamic Buckling Pressures

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Nomenclature

$\hat{H}$	=	magnetic induction
n	=	number of turns per unit length
$f(x, y, R, l)$	=	geometry function
R	=	radius of solenoid
x_p	=	coordinate along solenoid axis
l	=	length of solenoid
I_p	=	current in solenoid
C	=	discharge circuit capacity
Δ	=	width of search coil or specimen section
p	=	y/R
I_s	=	current in specimen
x	=	running coordinate along solenoid axis
y	=	running coordinate along solenoid radius
K	=	force
P	=	pressure
L	=	discharge circuit inductance
ρ_s	=	resistance of specimen
t	=	time
F	=	cross section of specimen
k_1	=	$(2x_p + l)/2R$
k_2	=	$(2x_p - l)/2R$

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Introduction

Current defense concepts for protection against ballistic war heads call for employment of nuclear blast to aid in the destruction of the attacking re-entry vehicle. Hence, effort must be directed toward simulation of the associated phenomena in the laboratory.

Because of the high closure rate between the blast front and the re-entry vehicle, the pressure pulse applied to the re-entry vehicle structure can be expected to have a duration in the μsec range. Shell design criteria for pressure pulses of this type are based on studies[1] accomplished with pressure pulses in the millisecond range. There is reason to question the applicability of these criteria, since the pulse duration is too long to excite the shell ring mode.

Investigations[2,3] using conventional explosives all lack that one ingredient so necessary in experimental studies, i.e., control. In addition, conventional explosives introduce hazards and instrumentation difficulties, which are not to be overlooked.

Lindberg[4] utilizes very thin shells to demonstrate the interaction between the elastic extensional mode and the elastic flexural modes. The use of such thin shells was rejected by the authors because of the difficulty of eliminating imperfections caused by fabrication and also because the possible existence of a relationship between the ring mode and breathing mode frequencies might preclude use of such specimens.

Experimental Techniques

A metallic shell is inserted into a coil through which a high-energy capacitor bank is discharged. The circuit is critically damped to avoid ringing. This discharge gives rise to a magnetic field, which, in turn, induces a current in the specimen. The inducing and induced currents generate magnetic fields that repel each other, thus generating the pressure pulse. Since it is possible to calculate the magnetic fields, the pressure can be determined. Both radially symmetric and asymmetric pulses can be generated by the simple expedient of varying the radial location of the specimen within the coil. Presently, studies are being conducted for devising a pressure gage on photoelastic principles. Although results are encouraging, satisfactory measurements have not yet been obtained. Space limitations prevent further elaborations.

Analytical Development

In the following, an analytic expression for the pressure will be derived. Consider the force acting on a current I_s in a magnetic field $\hat{H}$. It may be calculated from

$$dK = \hat{H} I_s \, dl \sin(\hat{H}, dl) \qquad (1)$$

In this case, only the axial component of the field is important, and $\sin(\hat{H}, dl)$ may be assumed to equal 1. For a cylindrical coil, $\hat{H}$ has been determined to be[7]

$$\hat{H}_{ax} = \frac{In}{R} \int_0^\pi \left[\frac{2x_p - l}{(1 + p^2 + k_2^2 - 2p\cos\gamma)^{1/2}} - \frac{2x_p + l}{(1 + p^2 + k_1^2 - 2p\cos\gamma)^{1/2}} \right] \frac{1 - p\cos\gamma}{(1 + p^2 - 2p\cos\gamma)} \, d\gamma \qquad (2)$$

To determine I_s, we compute the mean induced voltage in a ring segment of the specimen of width Δ at the distance $x_p + \Delta/2$ from the origin:

$$E\left(x + \frac{\Delta}{2}, t\right) = -\frac{2n}{\Delta}\frac{dI_p}{dt} \int_x^{x+\Delta} \int_F f(x, y, R, l) dF dx \qquad (3)$$

From (3), we obtain the current in the ring section as

$$I_s = -\frac{2n}{\Delta \rho s}\frac{dI_p}{dt} \int_x^{x+\Delta} \int_F f(x, y, R, l) \, dF dx \qquad (4)$$